

The Effects of Proof Features and Question Probing on Understanding Geometry Proof

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Abstract

This study aims to investigate how the written formats, complexity of proofs and the types of understanding questions affect students' understanding of geometry proof. Theoretically, Duval's three levels of organizing statements - micro, local and global, are applied to assess 153 ninth graders' understanding of geometry proof. The results show (a) there was no interaction among written formats, complexity of proofs, and types of understanding questions in terms of students' understanding of geometry proof; (b) local understanding is the easiest for students; (c) the effects of the complexity of proofs on local and global understanding were statistically

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significant. It is noted that the generalizability of the results is limited by the task of proof texts without their corresponding propositions. The factor mixing proof steps and familiarity of propositions should be taken into account while arranging learning sequence of reading proofs. Afterward, further research is proposed in this paper.

Keywords: geometry proof, text, understanding

幾何證明的文本特徵與提問類型對 學生閱讀理解表現的影響

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摘 要

本研究主要探討的問題是：不同寫法、不同複雜度和不同種類的理解問題對學生理解幾何證明有何影響？在理論上，採用Duval的組織敘述之3種層次做為不同種類的理解，依此設計工具測驗153位國三學生對幾何證明的理解。研究結果顯示：1.不同寫法、不同複雜度和不同種類的理解問題之間沒有交互作用；2.局部理解是最容易的；3.不同複雜度會影響學生在局部理解和整體理解問題的表現。以上結果的一般性仍受限在沒有給命題的證明文本之測驗情境。在安排閱讀幾何證明的學習序列時，編輯者應該要考慮混合證明步驟數和命題熟悉度的複雜度因子。最後，本文將會提出進一步的研究議題。

關鍵詞：幾何證明、文本、理解

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Mathematics Proof for All with the Perspective of Reading Comprehension

Learning to read and read to learn is necessary competency for modern citizens. Progressive knowledge is continuously and abundantly created and received with different media. Accordingly, reading is not only recognition of words and recall of their meaning, but also an active and constructive process between readers, media and contents. In the learning of mathematics, the role of reading had been expanded in the classrooms via exploratory studies and action research (e.g. Borasi & Siegel, 2000). Borasi and Siegel noticed that most of the existing literature about reading mathematics mainly focused on mathematics as a body of facts and techniques, and might fail to employ reading to support students' learning in mathematics classes. Although they argued that reading is beyond a set of skills for extracting information, "reading to comprehend" can not be prevented from being identified as a major way used to learn mathematics in classes. Extracting information to comprehend texts is necessary for both learning to read and reading to learn.

To study the learning of mathematics proof, some researchers have explored students' proof schemes (Harel & Sowder, 1998; Housman & Porter, 2003) or surveyed students' understanding of proofs (Healy & Hoyles, 2000). They found even high-attainers in mathematics embrace informal or non-scientific conceptions of proofs. In particular, Tall (1998) had argue that mathematical proof is for some not for all. However, we search for a broader view of learning proofs to help students engage in mathematics proofs, and to leave no child behind. Yang and Lin (2008) provided evidence to support that approaching geometry proof from reading or

connecting reading and writing may be better than from writing.

Students' goals of learning proofs in classroom might be different from mathematicians' goals of constructing proofs (Yang, Lin, & Wu, 2008). Mathematicians take proofs as both a tool and a product for creating new knowledge. On the contrary, most students may have difficulties in constructing formal proofs even after conjecturing by their reasoning. However, validating formal proofs have the potential contributions to their learning of mathematics proofs (Selden & Selden, 2003). While taking a perspective of reading literacy for learning proofs, Yang and Lin (2008) investigated a model of Reading Comprehension of Geometry Proof (RCGP) to elaborate which facets the text of geometry proof can be comprehended from. Lin and Yang (2007) further found that logical reasoning and relevant geometric knowledge accounted for 54% of the variance on RCGP data from the ninth graders who were going to learn formal geometry proof in school.

Influence of Text Feature on Reading Comprehension

From motivated by the construction-integration model of text comprehension (Kintsch, 1988), many studies investigated the complex interactions among learner, task and text factors which may influence different results of comprehension (Goldman, Varma, & Cote, 1996). Learners' good prior knowledge does not always favor good reading comprehension. McNamara, Kintsch, Songer and Kintsch (1996) found that readers who know little about the text benefit from the coherence of text, and that a minimally coherent text forces the knowledgeable readers to engage into inferring unstated relations underlying the text. Therefore, features of texts are factors which interact with readers' knowledge of texts.

Based on the schema-theoretic view, studies in language reading have reported the effects of familiarity with text content or structure on inference and comprehension (e.g. Lee, 1986; Ostler & Kaplan, 1982). For example, McGee (1982) found that the awareness of text structure was correlated with recall of important textual information. Readers identified main ideas and relations between ideas in well-presented physical text (Seidenberg, 1989). A collection/description text structure presented the least organization among comparison/contrast, problem/solution and cause/effect text structures and resulted in the poorest awareness and recall (Meyer & Freedle, 1984). Regarding specific content, the writing genre, proof-first or principle-first, is also a factor influencing comprehension of scientific texts. DeLucas and Larkin (1990) found that the proof-first structure is more difficult than the principle-first structure, and students tended to summarize the scientific texts with the principle-first structure whenever reading a proof-first or principle-first text.

Features of Geometry Proof in Textbooks and Different Language

While proof is viewed as a genre of mathematical text, it is expected to rethink the features of materials for benefiting students' RCGP in addition to the exploration of teaching methods or activities. Based on the features of proofs texts in mathematics textbooks, it can be found that the two-column format and the line-by-line format of proofs are most familiar. However, the line-by-line format of proofs is most popular in the present mathematics textbooks. It may result from that the two-column format is difficult for students to produce and understand, and dissociates the doing of proofs from the construction of knowledge (Herbst, 2002). Based on the

theoretical research which suggest that reading and writing rely on analogous mental processes and isomorphic knowledge (Fitzgerald & Shanahan, 2000), is learning to read proof with two-column format strange to our students, like learning to write?

In proof texts of the two-column format, proof steps were separated into conclusion left and reason right, as shown in figure 1. Accordingly, the information firstly caught is conclusion and then reason, which might be consistent with English conjunctive, e.g. He likes mathematics because he likes his mathematics teacher, but not consistent with Chinese convention, e.g. 他因為喜歡他的數學老師，所以喜歡數學. While students who are native speakers of Chinese are taught to read or write proofs with the two-column format, they may encounter difficulties of both linguistic and proof forms. Therefore, the place of reason, in the right or left column, may be a factor of influencing students' understanding of proofs.

Proof:	
Conclusion:	Reason:
1. $\angle BAC = \angle ACD$ and $\overline{AD} = \overline{BC}$.	1. $\text{Arc}(AD) = \text{Arc}(BC)$
2. $\overline{AB} \parallel \overline{DC}$	2. $\angle BAC = \angle ACD$ and the interior alternate angles are equal
3. $ABCD$ is an isosceles trapezium	3. $\overline{AB} \parallel \overline{CD}$ and $\overline{AD} = \overline{BC}$

Figure 1. A proof with the two-column format of conclusion left and reason right

Geometry proof is a topic in secondary mathematics. We find a common feature in mathematics textbooks in each country and in any times; that is, a proposition with few proof steps is always precedent to a proposition with many proof steps. If the contents of propositions is controlled, and to write proofs is the learning goal, propositions with few proof steps precedent to many proof steps make

sense. However, if the focus is shifted from writing proofs to reading proofs, and the familiarity with the contents of propositions is different, the sequence of different proofs is required to reconsider. A balance between substance and form of proofs appealed by Herbst (2002) should be taken into account while investigating students' RCGP with respect to different complexity of proof.

In recent two decades, Taiwan mathematics textbook presents proofs with line-by-line formats. Regarding informal discussion with some mathematics teachers, we found two reasons for using this format. One was that writing with the two-column format was time-consuming, and the other was that students did not really know why to write in this way and just memorize its format. Therefore, the line-by-line format substitutes the two-column format. In addition, proof with the line-by-line format in mathematics textbooks could be divided into two sub-types. One is the sub-type of claim antecedent to arguments; for example, the start of the proof is that to prove C , we must show that B , where B logically implies C , and the complete proof steps are as shown in figure 2. The other is the sub-type of arguments antecedent to the claim; for example, the start of the proof is the given, and the complete proof steps are as shown in figure 3. The first and the second sub-types are the backward and forward proof methods respectively. Although both of them are often adopted to construct a proof, a written proof is often presented with one type. How the two types of the line-by-line format affect readers' reading comprehension is still a question.

Proof:
 If $\overline{AB} \parallel \overline{CD}$ and $\overline{AD} = \overline{BC}$, then $ABCD$ is an isosceles trapezium.
 Because $\widehat{AD} = \widehat{BC}$, then $\angle BAC = \angle ACD$ and $\overline{AD} = \overline{BC}$.
 Because $\angle BAC = \angle ACD$, then $\overline{AB} \parallel \overline{DC}$ (the interior alternate angles are equal).

Figure 2. A proof with the line-by-line format of claim antecedent to arguments

Proof:
 Because $\widehat{AD} = \widehat{BC}$, then $\angle BAC = \angle ACD$ and $\overline{AD} = \overline{BC}$.
 Because $\angle BAC = \angle ACD$, then $\overline{AB} \parallel \overline{DC}$ (the interior alternate angles are equal).
 Because $\overline{AB} \parallel \overline{CD}$ and $\overline{AD} = \overline{BC}$, then $ABCD$ is an isosceles trapezium.

Figure 3. A proof with the line-by-line format of arguments antecedent to claim

Reading Comprehension of Geometry Proof

We further suspected that the effects of text features would depend on the kind of comprehension assessed. Lin and Yang (2007) has investigated students' performance on reading comprehension of geometry proof with the forward proof method regarding the line-by-line format. They found that students' reading comprehension of geometry proof might not develop from micro, local to global understanding. Students' performance in reading comprehension of geometry proof showed two different types, labeled as relational and instrumental comprehension. The relational comprehension denotes the understanding of geometry proof develops from surface knowledge, recognizing elements (premises, conclusions or applied properties), chaining elements to encapsulation (knowing the generality of a proposition and properly applying it). The instrumental comprehension denotes the understanding of geometry proof jumps from surface knowledge to beyond chaining elements and

then to come back to recognizing and chaining elements.

Their instrument of measuring students' reading comprehension of geometry proof was delicate, and answering a set of items for just one proof takes about thirty minutes. But comparing students' understanding of geometry proof in four types of the written formats and in two levels of the complexity of proofs, a modified instrument is required. On the other hand, proofs without their corresponding propositions are set as the context of the task for engaging students into active inference where organizing proof steps are required, which is different from the task of RCGP (Yang & Lin, 2008). Thus, Duval's (1998) three levels of organizing statements-micro, local and global are used to assess students' understanding of geometry proof. The three levels of organizing proof steps are to theoretically link a step, several steps and all steps in a proof respectively.

Already 85% of the Taiwanese students achieved the upper half of the international samples (Mullis, Martin, Gonzalez et al. 2000). They outperformed even for complex problem-solving items. Although the subjects did not learn geometry proof in school, 46.4% of grade 8 Taiwan students can judge correctly or at least produce the crucial elements for proving one-stage proof items, and 48.5% for multiple-stage proof items (Heinze, Cheng and Yang, 2004). Therefore, it is assumed that a proof without its corresponding proposition (ref. Yang & Wang, 2008) can be used to assess students' understanding of proofs based on the perspective of the reading approach to learning mathematics proof even if students have not learnt secondary geometry proof in school.

Research Question

It seems plausible that a proof with fewer steps is easier to understand than a proof with more steps. However, this plausible belief is unsure under the contrast between the familiar and unfamiliar propositions. Although National Council of Teachers of Mathematics (NCTM) (1989) deemphasized the written format of proofs, the role of the written formats, either two-column or line-by-line, may result in different effects on writing and reading performance. Therefore, this study investigated the effects of the three factors-the written formats, the complexity of proofs and the kinds of understanding questions-on students' understanding of geometry proof. In textbook, both a proposition and its proof are provided. Herein, the task of a proof without its corresponding proposition just presented a proof to students, and several questions are asked to assess students' understanding of geometry proof.

Accordingly, the research questions include (a) are there three-way and two-way interactions between the written formats, the complexity of proofs and the kinds of understanding questions, and (b) which written format, complexity, or kind of understanding questions favors students' understanding of geometry proof.

Method

The Study Instrument

In order to investigate the effects of the three factors-the written formats, the complexity of proofs and kinds of understanding questions-on students' understanding of geometry proof. The content of secondary geometry is adopted to design the

instrument. The written formats are mainly classified into the two-column format and the line-by-line format. Regarding the place of reasons and conclusion, two sub-types of the two-column format include reasons in the right column and reasons in the left column. Regarding the place of the claim presented in a proof, two sub-types of the line-by-line format include the claim antecedent to arguments and arguments antecedent to the claim. The number of proof steps and the familiarity of propositions are mixed to be a factor-the complexity of proofs-which includes three proof steps for an unfamiliar proposition and ten steps for a familiar proposition (Pythagorean Theorem). In sum, there are four types of the written formats for each proposition, and there are two levels of the complexity of proofs.

For each proof, three kinds of understanding questions are proposed, based on Duval's three levels of organizing statements-micro, local and global (Duval, 1998) to assess students' understanding of geometry proof. The first one requires students to identify the given and the claim, which denotes the micro understanding of geometry proofs. The second one requires students to logically chain several proof steps, which denotes the local understanding of geometry proofs. The last one requires students to comprehend several proof steps as a whole and appropriately apply it, which denotes the global understanding of geometry proofs. Although the terms, micro, local and global, are adopted from Duval's levels of organizing information, the meaning of micro, local and global understanding is limited to the questions designed in this study. It is noted that the three kinds of understanding questions do not imply the learning hierarchy from micro, local to global understanding. Moreover, the properties applied in each proof are provided and attached in the last part for the purpose of preventing students from losing their attention. An

exemplar of the study instrument is shown in figure 4, where question 1 to 3 belongs micro, local and global understanding respectively. Students would be guided to read the instruction before this proof, and then asked to write their answers to the following questions.

There are sixteen versions of the study instrument. Three proof steps for an unfamiliar proposition and ten steps for a familiar proposition (Pythagorean Theorem) are included in each version. Among four types of written formats, two different formats are assigned to the two propositions of each version. Thus, the number of permutation with repetition is sixteen.

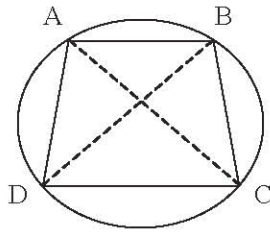
Subjects and Data Analysis

The instrument was administered to 155 ninth graders of four classes in one private junior high school, and just two of the subjects did not answer any question of the study instrument. These students who are about 14+ years old had learnt properties of congruent triangles, and are going to learn geometry proof regarding some simple geometrical figures. These students' activities in class mainly consist of practicing and memorizing mathematical concepts and procedures.

Lest students lose their patience, and a learning effect occurs during the test, each student is randomly assigned to answer one version of the study instrument where the two different propositions with two identical or difference formats are included. Thus, each kind of understanding questions for each proposition could be answered by about 38 subjects.

In this research, the dependent variables involved the correct rate of answers to each kind of understanding questions. Students' responses to the three kinds of

Below, there will be the steps of a proof, and the properties that are applied will be described after all of the steps. Please answer the questions and write the following answers in boxes.



Proof:

1. If " $\overline{AB} \parallel \overline{CD}$ and $\overline{AD} = \overline{BC}$ ", then "ABCD is an isosceles trapezium".
2. Because $\text{Arc}(AD) = \text{Arc}(BC)$, and properties (iii) and (ii), it can be derived that $\angle BAC = \angle ACD \dots *$ and $\overline{AD} = \overline{BC}$.
3. From * and property (i), what could you derive from there? (Please answer in Question 2).

Question 1:

- (1) Please describe the given of this proof and the claim of this proof.
- (2) If you would formulate a question to let your classmates write this proof, how would you word the question?

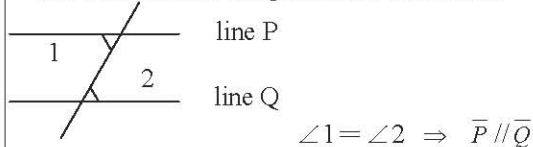
Question 2: What can you derive on the relation between \overline{AB} and \overline{CD} from Step 3? And write your reasons for this.

- (1) The relation between \overline{AB} and \overline{CD} is:
- (2) Reasons are:

Question 3: If $A'B'C'D'$ are on the same circle, and $\text{Arc}(A'B') = \text{Arc}(C'D')$, with reference of Steps 2 & 3, what could you derive from here?

Applied Properties:

- (i) If a line intersecting two line sections and the interior alternate angles are equal, the two line sections are parallel to each other.



- (ii) If two arc lengths are equal, then the corresponding chords will be equal, too.
- (iii) If two arc lengths are equal, then the corresponding circumference angles will be equal, too.

Figure 4. An exemplary item of three steps for an unfamiliar proposition with the line-by-line format where the claim is antecedent to arguments

questions for each level of the complexity of proofs were coded and then scored despite the types of the written formats. The principle of scoring is to differentiate the quality of students responses according the completeness and the optimization of given-claim, premise-applied property-conclusion, or applied conclusion. Therefore, the full mark of each kind of understanding questions was dependent on the varieties of responses. For example, students who answer $\text{Arc}(AD)=\text{Arc}(BC)$ and ABCD inscribed in a circle to question 1-1 (the given part) will get the full mark, 2. Students who answer $\angle D'A'C' = \angle A'C'B'$, $\overline{A'D'} // \overline{B'C'}$, or $\overline{A'B'} = \overline{C'D'}$ to question 3 (global) will get partial score, 1. The full marks of micro, local and global understanding questions are 6, 2, and 2 respectively. The intercoder reliability of two coders based on the answers of twenty randomly selected subjects was 93.3%, computed with the Miles and Huberman (1984) formula.

In order to compare students' understanding of geometry proof among the three factors, the score for each kind of understanding questions is divided by its full score, which is labelled as correct rate. The Statistical Package for the Social Sciences (SPSS 13.0 for Windows) was used for analyses. All the subjects would be tested by all kinds of understanding questions and both levels of complexity but two types of the written formats; thus, a three-way ANOVA mixed model with the two factors of complexity and understanding as repeated-measures factors is used to analyze the data of these subjects for the main effects of written format, complexity, understanding and their interactions. According to the result of the three-way ANOVA, simple main-effects analysis for understanding differences is conducted by repeated-measures one-way ANOVA, and simple main-effects analysis for complexity differences is conducted by paired t-test.

Result

No Interaction among Written Format, Complexity and Understanding

Table 1 presents the correct rates of answers to the micro, local and global understanding questions regarding the two levels of the complexity and the four types of the written formats. Regarding micro understanding questions, the correct rates range between 0.482 and 0.532 in the text of three proof steps for an unfamiliar proposition, and between 0.395 and 0.569 in the text of ten proof steps for a familiar proposition. Regarding local understanding questions, all of the correct rates in the text of three proof steps for an unfamiliar proposition are higher than those in the text of ten proof steps for a familiar proposition. On the contrary, all of the correct rates in the text of three proof steps for an unfamiliar proposition are lower than those in the text of ten proof steps for a familiar proposition regarding global understanding questions. Although the difference of students' correct rates between the two levels of complexity or between the four types of written formats is unclear, the correct rates of answers to local understanding questions is consistently higher than all the other rates of answers to micro and global understanding questions.

Moreover, a mixed design 4 (written formats) \times 2 (complexity of proofs) \times 3 (understanding kinds) analysis of variance was performed on correct rates with thirty seven students who were implemented to one version of the same written format. A significant two-way interaction effect of complexity by understanding, $F(2, 742.699)=14.798, p=0.000$, as well as a significant main effect of understanding, $F(2, 742.699)=83.707, p=0.000$, were found. Therefore, the four written formats

Table 1. Means and standard deviations of correct rates.

	Micro		Local		Global	
	Means	SD	Means	SD	Means	SD
Three proof steps for an unfamiliar proposition						
Written format						
line-by-line (the claim antecedent)	.482	.271	.803	.319	.395	.405
line-by-line (arguments antecedent)	.538	.302	.808	.295	.474	.428
two-column (reasons in the left)	.504	.317	.800	.295	.325	.446
two-column (reasons in the right)	.532	.238	.889	.270	.347	.393
Ten proof steps for a familiar proposition						
Written format						
line-by-line (the claim antecedent)	.395	.319	.675	.401	.533	.452
line-by-line (arguments antecedent)	.569	.345	.692	.423	.487	.432
two-column (reasons in the left)	.439	.344	.597	.444	.481	.481
two-column (reasons in the right)	.553	.306	.789	.361	.570	.490

did not result in difference of students' understanding of geometry proof. Regardless of the written formats, 42.5% and 36.0% of students' answers got 0 score in questions 1-(1-1) which asked students to identify the premise of the unfamiliar proposition with three proof steps and the familiar proposition with ten proof steps respectively. 30.7% and 43.8% of students' answers got 0 score in questions 1-(1-2) which asked students to identify the claim of the two levels of complexity of proofs respectively. 5.9% and 21.6% of students' answers got 0 score in questions 2 which asked students to identify the applied property of one argument in the two levels of complexity of proofs respectively. 49.0% and 35.3% of students' answers got 0 score in questions 3 which asked students to apply the propositions of the two levels of complexity of proofs respectively.

Local Understanding is the Easiest

To aid in the interpretation of the two-way interaction effect of complexity by understanding, tests for simple main effects of understanding kinds were firstly performed. Repeated-measures ANOVAs revealed a significant difference regarding both levels of complexity (three proof steps, $F(1.712, 260.263)=84.953$, $MSE=0.105$, $p=0.000$; ten proof steps, $F(1.918, 291.473)=24.311$, $MSE=0.075$, $p=0.000$). Post hoc tests showed that the correct rate of answers to local understanding questions is significantly higher than the other two correct rates of answers to micro and global understanding questions regarding both levels of complexity. Regarding the three proof steps for an unfamiliar proposition, the correct rate of answers to micro understanding questions is significantly better than to global understanding questions. However, the difference between the correct rates of answers to micro and global understanding questions was not statistically significant regarding the ten proof steps for a familiar proposition.

Diverse Complexity of Proofs Favors Different Kinds of Understanding

Subsequently, tests for simple main effects of complexity were performed. The results showed that the effects of complexity on the correct rates were statistically significant regarding local ($t(152)=3.646$, $p=0.000$) and global ($t(152)=-3.038$, $p=0.003$) understanding questions. However, there was no significant difference between the two levels of complexity regarding micro understanding questions ($t(152)=1.123$, $p=0.263$). Local understanding questions are easier in reading the three proof steps for an unfamiliar proposition than in reading the ten proof steps. On the contrary, global understanding questions are easier in reading the ten proof steps

for a familiar proposition than in reading the three proof steps for an unfamiliar proposition. Accordingly, diverse complexity of proofs may favor different kinds of understanding.

Conclusion And Suggestion

No significant interaction among the written formats by complexity of proofs by understanding kinds, and no significant interaction between the written formats by complexity implied that the effect of the written formats on each kind of understanding is consistent between the two levels of complexity of proofs. This consistency occurs in students' understanding of geometry proof, but does not necessarily extend to students' reading strategies. It seems that the strategy of structure analysis is more useful in the line-by-line format, and the strategy of remedy, where students read reasons while they do not understand its corresponding conclusions, is naturally used in the two-column format. Of course, this conjecture is required to verify by practical research.

The context of the task in this study is to present a proof without its corresponding proposition; therefore, the status of the information is implicit to readers. Theoretically, readers who first get the starting point from the argument-antecedent format should comprehend better than readers who first get the ending point from the claim-antecedent format, because it is hard to further identify the status of the statement followed the ending point if the claim-antecedent format is not identified by readers. However, no evidence supported this hypothesis in this study. Further survey with larger samples is required in order to increase the power of the test, which may consist with the theoretical conjecture.

It is unexpected that the reason-in-the-right format benefits our subjects whose native language is Chinese and Taiwanese language. However, figures, the learning of English and daily conversation may affect students' understanding of geometry proof in addition to the syntactic structure of language. Regarding figural inference, it is helpful to see the underlying properties where the warrant or backing is still implicit (Cheng & Lin, 2007). Therefore, the students did not encounter more difficulty in reading the reason-in-the-right format. The subjects in this study had learnt English in school for two years, and we also tell others the result and subsequently the reasons in our daily conversation. Thus, the relevant experience may cause no significant difference in the understanding of geometry proofs between the reasons-in-the-right format and the reasons-in-the-left format.

Although the effect of the written formats on the three kinds of understanding is not obvious in this study, it may occur while different contexts of proof texts, such as inserting invalid arguments or missing partial proof steps, and different domains of reading comprehension, such as recalling, summarizing or extending are regarded. Therefore, it requires further study to test if the two-column format benefits students to identify the logical status and to logically chain arguments while the status of arguments is explicit and formed, and if the line-by-line format benefits students to find critical proof ideas while each sub-step is followed with its applied properties.

On the other hand, the complexity of proofs makes difference in local and global understanding of geometry proof. An unfamiliar proposition with fewer proof steps benefits local understanding, and a familiar proposition with more proof steps benefits global understanding. This tells us that different teaching sequences should be arranged for understanding proofs of familiar and unfamiliar propositions where

different length of proof steps is taken into account. For advancing students' understanding of geometry proof, the local understanding questions could be discussed with fewer proof steps, and the global understanding questions could be discussed with familiar propositions. For example, students can be guided to find applied properties while reading proofs of unfamiliar propositions with fewer proof steps. On the contrary, students can be guided to apply the familiar proposition to infer some conclusion from a similar premise while reading proofs of familiar propositions with more proof steps.

The result that local understanding questions are easiest is a little inconsistent with Yang and Lin's (2008) model of reading comprehension of geometry proof. It might result from the different instruments in the two studies. In this study, the corresponding proposition was not provided, which obviously resulted in students' difficulty in micro understanding. That is, micro understanding is easier than local and global understanding while the proposition is provided. Regarding each kind of understanding, it is surly that familiar propositions with fewer proof steps are easiest to understand; therefore, all kinds of understanding can be discussed while learning to read fewer proof steps of familiar propositions. However, familiar propositions with fewer proof steps may not drives students to see the necessity of using appropriate reading strategies, which requires to learn advanced mathematics or evaluate peers' flawed proofs.

The issues about the effects of text features on understanding of geometry proof are worthy of further research while it is criticized that the proofs in textbooks is arranged from proof with fewer steps to proof with more steps regarding all kinds of comprehension, and that the line-by-line format is better than the two-column format

for leaning mathematics proof. Besides, to initiate systematic projects on the influences of nonlinear text in the learning of geometry proof is required for the technology-based learning environment.

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References

- Borasi, R., & Siegel, M. (2000). *Reading counts: Expanding the role of reading in mathematics classrooms*. New York: Teachers College Press.
- Cheng, Y. H., & Lin, F. L. (2007). The effectiveness and limitation of reading and coloring strategy in learning geometry proof. In J. H. Woo, K. S. Park & D. Y. Seo (Eds.), *Proceedings of the 31st PME Conference*, 2, 113-120.
- Dee-Lucas, D., & Larkin, J. H. (1990). Organization and comprehensibility in scientific proofs, or "Consider a particle". *Journal of Educational Psychology*, 82(4), 701-714.
- Duval, R. (1998). Geometry from a cognitive point of view. In C. Mammana & V. Villani (Eds.), *Perspectives on the Teaching of Geometry for the 21st century. An International Commission on Mathematical Instruction (ICMI) Study* (pp. 37-52). Boston, MA: Kluwer Academic Publisher.
- Goldman, S., Varma, S., & Cote, N. (1996). Extending capacity-constrained construction integration: Toward "smarter" and flexible models of text comprehension. In B. K. Britton & A. C. Grasser (Eds.), *Models of Understanding Text* (pp. 73-114). Mahwah, NJ: Lawrence Erlbaum Associate.

- Harel, G., & Sowder, L. (1998). Student's proof schemes: Results from exploratory studies. In A. H. Schoenfeld, J. Kaput & E. Dubinsky (Eds.), *Research on Collegiate Mathematics Education III* (pp. 234-283). Providence, RI: American Mathematical Society.
- Healy, L., & Hoyles, C. (2000). A Study of Proof Conceptions in Algebra. *Journal for Research in Mathematics Education*, 31(4), 396-428.
- Heinze, A., Cheng, Y. H., & Yang, K. L. (2004). Students' performance in reasoning and proof in Taiwan and Germany: Results, paradoxes and open questions. *Zentralblatt für Didaktik der Mathematik(ZDM)*, 36(5), 162-171.
- Herbst, P. (2002). Establishing a custom of proving in American school geometry: Evolution of the two-column proof in the early twentieth century. *Educational Studies in Mathematics*, 49, 283-312.
- Housman, D., & Porter, M. (2003). Proof schemes and learning strategies of above-average mathematics students. *Educational Studies in Mathematics*, 53(2), 139-158.
- Kaplan, R. B., & Ostler, S. (1982, May). *Contrastive rhetoric revisited*. Paper presented at the 16th Annual TESOL Convention, Honolulu, Hawaii.
- Kintsch, W. (1988). The use of knowledge in discourse processing: A construction-integration model. *Psychological Review*, 95, 163-182.
- Lee, J. F. (1986). Background knowledge and L2 reading. *Modern Language Journal*, 70, 351-354.
- Lin, F. L., & Tsao, L. C. (1999). Exam maths re-examined. In C. Hoyles, C. Morgan & G. Woodhouse (Eds.), *Rethinking the Mathematics Curriculum* (pp. 228-239). London: Falmer Press.
- Lin, F. L., & Yang, K. L. (2007). The reading comprehension of geometric proofs: The contribution of knowledge and reasoning. *International Journal of Science and Mathematics Education*, 5(4), 729-754.
- McGee, L. M. (1982). Awareness of text structure. *Reading Research Quarterly*, 17, 581-590.
- McNamara, D. S., Kintsch, E., Songer, N. B., & Kintsch, W. (1996). Are good texts always better? Text coherence, background knowledge, and levels of understanding in learning from text. *Cognition and Instruction*, 14, 1-43.
- Meyer, B. J. F., & Freedle, R. O. (1984). Effects of Discourse Type on Recall. *American*

- Educational Research Journal*, 21(1), 121-143.
- Miles, M., & Huberman, M. (1984). *Qualitative data analysis: A sourcebook of new methods*. Beverly Hills, CA: Sage.
- Mullis, I. V. S., Martin, M. O., Gonzalez, E. J., Gregory, K. D., Garden, R. A., O'Connor, K. M., Chrostowski, S. J., & Smith, T. A. (2000). *TIMSS 1999 International Mathematics Report*. IEA. Boston: International Study Center Lynch School of Education.
- National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: Author.
- Seidenberg, P. L. (1989). Relating text-processing research to reading and writing instruction for learning disabled students. *Learning Disabilities Focus*, 5(1), 4-12.
- Selden, A., & Selden, J. (2003). Validations of proofs considered as texts: Can undergraduates tell whether an argument proves a theorem? *Journal for Research in Mathematics Education*, 34(1), 4-36.
- Tall, D. (1998, August). *The cognitive development of proof: Is mathematical proof for all or for some?* Paper presented at the conference of the University of Chicago School Mathematics Project, Chicago, IL.
- Yang, K. L., and Lin, F. L. (2008). A model of reading comprehension of geometry proof. *Educational Studies in Mathematics Education*, 67(1), 59-76.
- Yang, K. L., & Wang, L. W. (2008, July). *Propositions posed under a proof without its proposition*. Paper accepted to present at the 11th International Congress on Mathematical Education, Monterrey, Mexico.
- Yang, K. L., Lin, F. L., & Wu, J. D. (2008). *Re-investigating Characteristics of Mathematical Conjecturing*. Paper submitted to PME 32, Mexico.